長庚大學 106 學年度第一學期電機所博士班演算法資格考

1. Please write down your student ID and name on the answer sheet.
2. Please indicate the number of each your answer that is relative to the problem.
3. Any form of cheating will lead to fail.

Please select five problems to answer. Total score of this exam is 100. Maximum deduction of 20 points for each problem that your answer.

1. If |V| is the number of vertices in the directed graph, and |E| is the number of edges, what is the running time of Dijkstra's algorithm in O( ) notation? Give a brief justification for your answer. You should also briefly describe any assumptions you are making about the implementation that would affect the answer.
2. If we modify the RELAX portion of the Bellman-Ford algorithm so that it updates d[v] and π[v] if $d[v] \geq d[u] + w(u, v)$ (instead of doing so only if d[v] is strictly greater than d[u] + w(u, v)), does the resulting algorithm still produce correct shortest-path weights and a correct shortest-path tree? Justify your answer.
3. Given a list of n integers, $v_1, \ldots, v_n$, the *product-sum* is the largest sum that can be formed by multiplying adjacent elements in the list. Each element can be matched with at most one of its neighbors. For example, given the list 1, 2, 3, 1 the product sum is $8 = 1 + (2 \times 3) + 1$, and given the list 2, 2, 1, 3, 2, 1, 2, 2, 1, 2 the product sum is $19 = (2 \times 2) + 1 + (3 \times 2) + 1 + (2 \times 2) + 1 + 2$.

   (a) Compute the product-sum of 1, 4, 3, 2, 3, 4, 2

   (b) Give the optimization formula for computing the product-sum of the first *j* elements

   (c) Give a dynamic program for computing the value of the product sum of a list of integers.
4. (a) Two of the most common divide-and-conquer sorting algorithms are quicksort and mergesort. In practice quicksort is often used for sorting data in main storage rather than mergesort. Give a reason why quicksort is likely to be the preferred sorting algorithm for this application; (b) Quicksort's worst-case running time is $O(n^2)$, but it has an expected running time of O(nlogn) if the partition function works well. What needs to be true about the partition function in order for the running time to be O(nlogn)? In practice, how can we ensure that this happens?
5. Consider the following recursive algorithm.

   ```
   public static int f(int n, int r) {
       if(r == 0 || r >= n) return 1;
       else return f(n - 1, r - 1) + f(n - 1, r);}
   ```

   Using dynamic programming, convert this into a method that takes O(n · r) time.
6. The longest common subsequence (LCS) problem is the problem of finding the longest subsequence common to all sequences in a set of sequences (often just two sequences). It is a classic computer science problem: the basis of data comparison programs such as the diff utility, and has applications in bioinformatics.

   (a) Use a dynamic programming algorithm to find the longest common subsequence between the two sequences: X = AGACATA; Y = GTACAAT ; (b) Based on (a), use the dynamic programming table determined for the above two sequences to determine the Longest Common Subsequence for X' = AGACA; Y' = GTACAAT
7. Let *f* be a flow in flow network G with source s and sink t, and let (S, T) be any cut of G. Then the net flow across (S, T) is $f(S, T)=|f|$. Please proof the description above.